CONDITIONS FOR REGULAR INTERACTION OF WEAK SHOCK WAVES

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Problems of interaction of weak shock waves arise in many physical phenomena and also, for example, in the study of interaction of shock waves due to explosion of two charges or due to two fast flying projectiles.

Problems of this type are very closely related to problems of reflection of weak shock waves. These investigations, as a rule, rely on utilization of theory of short waves developed in papers by Grib, Ryzhov and Khristianovich [1 and 2].

Utilizing basic premises of the theory of short waves, in this paper conditions for regular interaction of shock waves are examined, basic flow parameters are determined at the point of interaction, and the critical relationship for initial values (of intensity and of interaction angle of shock waves) is found which characterizes conditions for regular interaction. For the value of overpressure at the point of interaction, the surface of dependence on initial values is constructed. In case of symmetrical interaction results of this investigation coincide with data of regular reflection of weak shock waves from a rigid wall [3].

1. We shall examine the case of a regular interaction of weak shock waves. Let two planar shock waves OK_1 and OK_2 with overpressure p_1 and p_2 propagate in a quiescent medium and let them intersect at a small angle α. To this end the point of intersection of shock waves describes trajectory RO with time. The origin of the cylindrical system of coordinates r, ϑ is placed at point R, the point of initial interaction of the shock waves. The axis $\vartheta = 0$ is directed along the tangent to the trajectory. For a sufficiently large interval of time t the pattern of penetration (Fig.1) will consist of incident fronts OK_1, and OK_2 with constant overpressure p_1 and p_2 and reflected fronts OB_1 and OB_2 where overpressure decreases from some value p_0 at the point O to p_1 and p_2 at points B_1 and B_2 respectively.

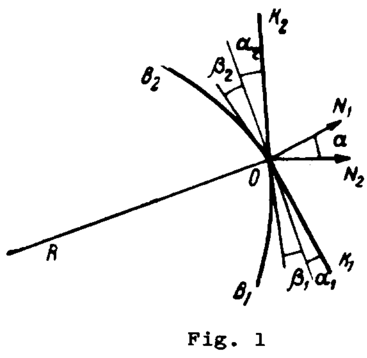

Fig. 1

In the case $p_1 = p_2$ this gives a pattern of symmetrical penetration which is analogous to regular reflection of shock waves from a rigid wall. The angles α_1 and α_2 (between the normal to the trajectory of motion at the point O and the shock fronts) will be referred to as angles of incidence and β_1 and β_2 (between the same normal and the tangents to the reflected fronts) will be called the angles of reflection.

The constant intensity of shock waves p_1 and p_2 with respect to time, the trajectory of motion of the point of interaction will be rectilinear and the entire pattern of penetration self-similar. It is therefore convenient for investigation of the flow in the vicinity of point O to go, as was done in the theory of short waves [1 and 2], over to nondimensional functions of velocity μ and ν and the coordinates δ, Y and τ which are related to the velocities and coordinates of the cylindrical system through the following relationships:

$$\mu = \frac{M}{M_0} = \frac{1}{M_0}\frac{u}{a_0}, \qquad \nu = \frac{1}{M_0\sqrt{{}^1/_2(n+1)M_0}}\frac{v}{a_0} \tag{1.1}$$

$$\delta = \frac{1}{{}^1/_2(n+1)M_0}\left(\frac{r}{a_0 t} - 1\right), \quad Y = \frac{\vartheta}{\sqrt{{}^1/_2(n+1)M_0}}, \quad \tau = \ln t, \quad M = \frac{p}{nP_0}$$

Here P_0 and a_0 are the pressure and velocity of sound in the quiescent medium, respectively, n is the constant ratio of specific heats (for air $P_0 = 1$ atm and $n = 1.4$), M_0 is some characteristic value of the quantity . In the case under examination M_0 is equal to the value of M at the point of intersection O.

At shock fronts, conditions for the normal component of velocity $M = p/nP_0$ and conditions for conservation of the tangential component of velocity in going through the front of the type [3]

$$u\psi - v = u'(\psi + \vartheta + \alpha') \tag{1.2}$$

will be fulfilled.

Here α' is the angle between the direction of flow velocity q' ahead of the wave front and the axis $\vartheta = 0, \psi$ is the angle between the normal to the shock front and the direction of the radius vector. The value of angle ψ is determined from the propagating equation of the shock wave front

$$\partial r / \partial t = N(1 + {}^1/_2\psi^2)$$

(N is the velocity of the front) through the relationship

$$\psi = \sqrt{{}^1/_2(n+1)M_0}\sqrt{2\delta - (\mu + \mu')} \tag{1.3}$$

2. Boundary conditions (1.2) at the shock fronts OB_1 and OB_2 take the form

$$u\psi_1 + v = u_1(\psi_1 + \alpha_1 - \vartheta), \qquad u\psi_2 - v = u_2(\psi_2 + \alpha_2 + \vartheta) \tag{2.1}$$

In terms of notation (1.1) relationships (1.3) and (2.1) when applied to the shock fronts OK_1 and OK_2 yield

$$\alpha_1 - \vartheta = \sqrt{{}^1/_2(n+1)M_0}\sqrt{2\delta - \mu_1}, \qquad \alpha_2 + \vartheta = \sqrt{{}^1/_2(n+1)M_0}\sqrt{2\delta - \mu_2} \tag{2.2}$$

and on fronts OB_1 and OB_2 they yield conditions

$$(\mu - \mu_1)\psi_1 + \nu\sqrt{{}^1/_2(n+1)M_0} = \mu_1(\alpha_1 - \vartheta), \quad \psi_1 = \sqrt{{}^1/_2(n+1)M_0}\sqrt{2\delta - (\mu + \mu_1)} \tag{2.3}$$

$$(\mu - \mu_2)\psi_2 - \nu\sqrt{{}^1/_2(n+1)M_0} = \mu_2(\alpha_2 + \vartheta), \quad \psi_2 = \sqrt{{}^1/_2(n+1)M_0}\sqrt{2\delta - (\mu + \mu_2)} \tag{2.4}$$

3. We shall investigate the flow in the vicinity of point O. Since M_0 is the value of M at point O, then $\mu_0 = 1$. Conditions (2.2), (2.3) and (2.4) on shock fronts take the following form in point O

$$\alpha_1 = \sqrt{{}^1/_2(n+1)M_0}\sqrt{2\delta_0 - \mu_1}, \qquad \alpha_2 = \sqrt{{}^1/_2(n+1)M_0}\sqrt{2\delta_0 - \mu_2} \tag{3.1}$$

$$(1 - \mu_1)\beta_1 + \nu_0\sqrt{{}^1/_2(n+1)M_0} = \mu_1\alpha_1, \qquad \beta_1 = \sqrt{{}^1/_2(n+1)M_0}\sqrt{2\delta_0 - (\mu_1 + 1)} \tag{3.2}$$

$$(1 - \mu_2)\beta_2 - \nu_0\sqrt{{}^1/_2(n+1)M_0} = \mu_2\alpha_2, \qquad \beta_2 = \sqrt{{}^1/_2(n+1)M_0}\sqrt{2\delta_0 - (\mu_2 + 1)} \tag{3.3}$$

In this connection

$$\alpha_1 + \alpha_2 = \alpha \tag{3.4}$$

Combining conditions (3.1) the following expression for δ_0 is obtained:

$$\delta_0 = \frac{2\alpha^2}{2M_0}, \qquad a = \frac{{}^1/_4\,[a^2/(n+1) + M_1 + M_2]^2 - M_1 M_2}{2\alpha^2/(n+1)} \tag{3.5}$$

while M_1 and M_2 are determined according to (1.2) through p_1 and p_2 .

For angles of interation $\alpha_1, \alpha_2, \beta_1, \beta_2$ and velocity ν_0 we now have Expressions

$$\alpha_1 = \sqrt{{}^1/_2(n+1)(a - M_1)}, \qquad \alpha_2 = \sqrt{{}^1/_2(n+1)(a - M_2)}$$

$$\beta_1 = \sqrt{{}^1/_2(n+1)(a - M_1 - M_0)}, \qquad \beta_2 = \sqrt{{}^1/_2(n+1)(a - M_2 - M_0)} \tag{3.6}$$

$$\nu_0 = (1 - M_2/M_0)\sqrt{(a - M_2 - M_0)/M_0} - (M_2/M_0)\sqrt{(a - M_2)/M_0}$$

Finally, combining the first conditions(3.2) and (3.3) we obtain for M_0 the following equation:

$$AM_0^3 + BM_0^2 + CM_0 + D = 0 \tag{3.7}$$

with coefficients

$$A = (M_1 - M_2)^2$$

$$B = 8aM_1M_2 - 2(M_1 + M_2)(M_1^2 + M_2^2) + 8M_1M_2\sqrt{(a - M_1)(a - M_2)} \tag{3.8}$$

$$C = (M_1^2 - M_2^2)^2 + 4M_1^2M_2[M_1 - \sqrt{(a - M_1)(a - M_2)}] +$$
$$+ 4M_1M_2^2[M_2 - \sqrt{(a - M_1)(a - M_2)}] - 8aM_1M_2[a + \sqrt{(a - M_1)(a - M_2)}]$$

$$D = 4M_1M_2[(M_2^2 - 2aM_1)(M_1 - \sqrt{(a - M_1)(a - M_2)}) +$$
$$+ (M_1^2 - 2aM_2)(M_2 - \sqrt{(a - M_1)(a - M_2)})] + 8aM_1M_2[a(M_1 + M_2) - M_1M_2]$$

We shall find the condition for initial parameters M_1, M_2 and α for which M_0 or, which is the same thing, the pressure at the point of interaction has the maximum value.

For Equation (3.7) we have $F(M_1, M_2, \alpha, M_0) = 0$; eliminating M_0 from the system $F'_{M_0}(M_1, M_2, \alpha, M_0) = 0$, $F(M_1, M_2, \alpha, M_0) = 0$, we obtain

$$\frac{K^3 - B^3}{A(3AD - BC)} = \frac{9}{2} \qquad (K = \sqrt{B^2 - 3AC}) \tag{3.9}$$

In this connection

$$M_0 = \frac{-B + \sqrt{B^2 - 3AC}}{3A} \tag{3.10}$$

Thus, for the range of regular interaction the initial parameters M_1, M_2 and α must be such that the left-hand side (3.9) will be greater or equal to 9/2.

4. For the case of symmetrical penetration of shock waves, when $M_1 = M_2$, Equation (3.7) transforms into a second order equation

$$M_0^2 - \left[\frac{2}{n+1}\left(\frac{\alpha}{2}\right)^2 + 2M_1\right]M_0 + \tag{4.1}$$
$$+ \left[\frac{4}{n+1}\left(\frac{\alpha}{2}\right)^2 + M_1\right]M_1 = 0$$

with the solution

$$\frac{M_0}{M_1} = \frac{1}{2}\left(\frac{\alpha^\vee}{2}\right)^2 + 1 - \frac{1}{2}\left(\frac{\alpha^\vee}{2}\right)\left(\left(\frac{\alpha^\vee}{2}\right)^2 - 4\right)^{1/2}$$

$$\alpha^\vee = \frac{\alpha}{\sqrt{{}^1/_2(n+1)M_1}} \tag{4.2}$$

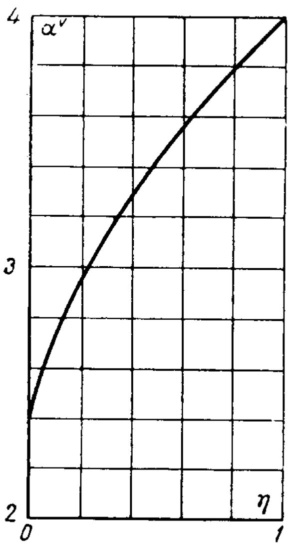

Fig. 2

The critical relationship (3.9) in this connection takes the form $\frac{1}{2}\alpha^{\vee} = 2$ i.e. we have flow conditions analogous to regular reflection from a rigid wall [1 and 3].

5. For examination of Equation (3.7) we introduce the parameters

$$\eta = \frac{M_2}{M_1}, \qquad \alpha^{\vee} = \frac{\alpha}{\sqrt{{}^1/_2 (n+1) M_1}} \tag{5.1}$$

Then, for the value of the relative overpressure in the point of interaction we obtain the following equation:

$$A^\circ \left(\frac{M_0}{M_1}\right)^3 + B^\circ \left(\frac{M_0}{M_1}\right)^2 + C^\circ \left(\frac{M_0}{M_1}\right) + D^\circ = 0 \tag{5.2}$$

with coefficients

$$\begin{aligned} A^\circ &= (1-\eta)^2, \; B^\circ = 2\,(2\alpha^{\vee 2}\eta - 1 + \eta + \eta^2 - \eta^3) \\ C^\circ &= \alpha^{\vee 4}\eta - 4\alpha^{\vee 2}\eta\,(1+\eta) + 1 + \eta - 4\eta^2 + \eta^3 + \eta^4 \\ D^\circ &= \eta\,[\alpha^{\vee 4}(1+\eta) + 4\alpha^{\vee 2}\eta - 1 + \eta + \eta^2 - \eta^3] \end{aligned} \tag{5.3}$$

The critical relationship (3.9)

$$4\,(3A^\circ C^\circ - B^{\circ 2}) + [9A^\circ(3A^\circ D^\circ - B^\circ C^\circ) + 2B^{\circ 3}]^2 = 0 \tag{5.4}$$

on substitution of coefficients (5.3) takes the form

$$\begin{aligned} &4\eta\,(\eta+1)^2\alpha^{\vee 12} - 8\eta\,(\eta+1)\,(5\eta^2 + 6\eta + 5)\,\alpha^{\vee 10} + (\eta-1)^2 \\ &(\eta^4 + 116\eta^3 + 214\eta^2 + 116\eta + 1)\,\alpha^{\vee 8} - 8\,(\eta-1)^4(\eta+1)\,(\eta^2 + 17\eta + 1)\,\alpha^{\vee 6} + \\ &+ 2\,(\eta-1)^6(7\eta^2 + 36\eta + 7)\,\alpha^{\vee 4} - 8\,(\eta-1)^8(\eta+1)\,\alpha^{\vee 2} + (\eta-1)^{10} = 0 \end{aligned} \tag{5.5}$$

Solution (3.10) on the critical multiformity (5.5)

$$\frac{M_0}{M_1} = \frac{-B^\circ + \sqrt{B^{02} - 3A^0C^0}}{3A^0} \tag{5.6}$$

takes the form

$$\frac{M_0}{M_1} = \frac{2\,[(\eta+1)(\eta-1)^2 - 2\alpha^{\vee 2}\eta]}{3\,(1-\eta)^2} + \tag{5.7}$$

$$+ \frac{\sqrt{(\eta-1)^4(\eta^2-\eta+1) - 4\alpha^{\vee 2}\eta\,(\eta-1)^2(\eta+1) + \alpha^{\vee 4}\eta\,(\eta+3)(3\eta+1)}}{3\,(1-\eta)^2}$$

By virtue of symmetry of Equation (3.7) with respect to parameters M_1 and M_2 Equations (5.2), (5.5) and (5.7) are invariant with respect to transformation

$$\eta_2 = \frac{1}{\eta_1}, \qquad \frac{M_0}{M_2} = \frac{M_0}{M_1}\frac{1}{\eta_1}, \qquad \alpha_2^{\vee} = \alpha_1^{\vee}\frac{1}{\sqrt{\eta_1}} \qquad \left(\eta_1 = \frac{M_2}{M_1}\right) \tag{5.8}$$

i.e. for Equations (5.2), (5.5) and (5.7) it is sufficient to examine solutions for

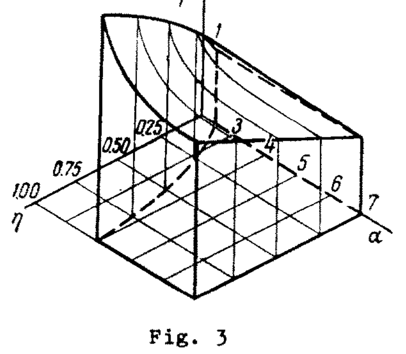

Fig. 3

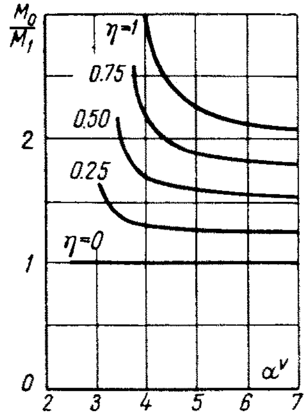

Fig. 4

η in the range $0 \leqslant \eta_1 \leqslant 1$; values for $1 \leqslant \eta_2 \leqslant \infty$ will be expressed through these solutions by Equations (5.8).

In Fig. 2 the curve of numerical solution of Equation (5.5) is presented which according to (4.3) goes through $\alpha^\vee = 4$ at $\eta = 1$. In this connection we have the following solution for Equation (5.5) in the vicinity of $\eta = 1$

$$\alpha^\vee = \left[\frac{2(5\eta^2 + 6\eta + 5)}{(\eta + 1)}\right]^{1/2} \tag{5.9}$$

For $\eta = 0$, the solution takes the form

$$\alpha^\vee = \sqrt{3 + 2\sqrt{2}} \qquad \text{for } \eta = 0$$

and for $\eta \to \infty$, according to (5.8), the solution approaches to

$$\alpha^\vee = \sqrt{(3 + 2\sqrt{2})}\,\eta \qquad \text{for } \eta \to \infty$$

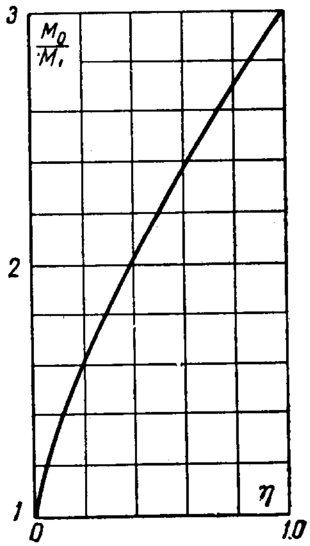

Fig. 5

In Fig.3 the surface of the solution M_0/M_1 of Equation (5.2) is constructed. Separate sections of this surface by planes η = const are presented in Fig.4, For $\eta = 0$ we have a constant value $M_0/M_1 = 1$ which corresponds to the case of zero intensity of the wave OK_2. For $\eta = 1$ we obtain the known solution (4.2) for symmetrical regular interaction [1 and 3]. For $\eta \to \infty$ the solution, according to (5.8), tends to the cylindrical surface $M_0/M_1 = \eta$ which corresponds to the case of zero intensity of the wave OK_1. In Fig.5, the form of the section of the solution surface by the cylindrical surface which passes through the critical multiformity parallel to the axis M_0/M_1, is given along axis $\alpha^\vee$.

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